



Solutions to large B and L breaking in the Randall–Sundrum model

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Abstract

The stability of proton and neutrino masses are discussed in the Randall–Sundrum model. We show that relevant operators should be suppressed, if the hierarchical Yukawa matrices are explained only by configurations of wavefunctions for fermions and the Higgs field along the extra dimension. We assume a Z_N discrete gauge symmetry to suppress those operators. In the Dirac neutrino case, there is an infinite number of symmetries which may forbid the dangerous operators. In the Majorana neutrino case, the discrete gauge symmetries should originate from $U(1)_X$ gauge symmetries which are broken on the Planck brane. We also comment on the $n - \bar{n}$ oscillation as a phenomenon which can distinguish those discrete gauge symmetries.

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1. Introduction

Models with low scale quantum gravity [1,2] have been intensely studied, since they can account for the hierarchy between the electroweak and the fundamental scales. However, in the low cut-off theories, higher dimensional operators induce in general fast proton decays or too large neutrino masses, since these operators are suppressed only by the low cut-off scale ~ 1 TeV.¹ It has been argued that these problems can be solved by some symmetries [4] or field configurations along the extra dimensions [5].

The solution by field configurations along the extra dimensions is interesting, since it also explains the hierarchical Yukawa matrices without introducing extra spectator fields [5–7]. In fact, there are some attempts in higher dimensional models to explain the hierarchical Yukawa matrices [8–10] and sufficiently long proton lifetime [5] by field configurations along the extra dimensions. However, these benefits do not go together in general, since the observed fermion masses require sufficient overlaps of wave functions for fermions, while the long proton lifetime requires small overlaps. For example, the proton lifetime is too short in the Randall–Sundrum (RS) model, if the observed fermion masses are explained by particular field configurations [9].

The purpose of this Letter is to study these problems in the RS model. In the next section, we analyze the proton lifetime and neutrino masses. We show that relevant operators must be suppressed substantially by hand, if the hierarchical Yukawa matrices are explained only by field configurations. In Section 3, we look for symmetries which may suppress those operators naturally. Here, we focus on discrete gauge symmetries [11],² since global symmetries may be explicitly broken by the topological effects of gravity [14], and there is no continuous gauge symmetry which suppresses the dangerous operators. We introduce a Z_N discrete gauge symmetry to suppress those operators. In the Dirac neutrino case, there is an infinite number of symmetries which may forbid the dangerous operators. In the Majorana neutrino case, discrete gauge symmetries cannot forbid the dangerous operators.

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¹ For other constraints (from flavor changing neutral currents, precision electroweak measurements, etc.), see [3] and references therein.

² Discrete gauge symmetries in extra dimension models have been studied in [12]. However, their model is different from ours in some ways. For example, right-handed neutrinos do not acquire large Majorana masses in their model. In our model, they acquire large Majorana masses and the usual seesaw mechanism [13] is realized, as we will show in Section 3.2.

However, if the discrete gauge symmetries originate from $U(1)_X$ gauge symmetries, and they are broken on the Planck brane, then the dangerous operators can be suppressed. Finally, we comment on $n - \bar{n}$ oscillation as a phenomenon which distinguishes between those discrete gauge symmetries.

2. Higher dimensional operators in the RS model

In some extra dimension models, the configurations of fermions and the Higgs field along the extra dimensions are employed to explain the proton stability, neutrino masses, and the hierarchical Yukawa matrices. However, in the RS model, the field configurations by themselves cannot explain the above issues simultaneously, as we will show in this section.

First, we summarize our setup. The metric of the RS model is

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (1)$$

where $\sigma = k|y|$, and $k \sim M_P = 2.4 \times 10^{18}$ GeV is the AdS curvature. The fifth dimension y is compactified on an orbifold S^1/Z_2 . Two 3-branes reside at the fixed points $y = 0, y = \pi R$, which are referred to as the Planck brane and the TeV brane, respectively. The only scale that appears in this model is the Planck scale: all terms in the 5D action are characterized by M_P . The effective scale on the TeV brane is $M_T \equiv e^{-\pi k R} M_P$. We take $M_T \sim 10$ TeV, that is, $kR \sim 10$ in the following arguments. We put the Higgs field H on the TeV brane to solve the hierarchy problem. The 5D Dirac fermions $\Psi_{qi}, \Psi_{ui}, \Psi_{di}, \Psi_{li}$ and Ψ_{ei} are in the bulk, and their chiral zero modes $q_{Li} = (u_{Li}, d_{Li}), u_{Ri}, d_{Ri}, l_{Li} = (v_{Li}, e_{Li})$ and e_{Ri} are the quarks and leptons of the Standard Model. The lower suffixes $i = 1, 2, 3$ denote the generations of fermions. The configurations of fermions explain the hierarchical Yukawa matrices [7].

The operators which generate 4D Majorana neutrino masses are

$$\int d^4x \int dy \sqrt{-g} \frac{1}{M_P^2} H H \bar{\Psi}_{li}^c \Psi_{lj} \delta(y - \pi R) \equiv \int d^4x (m_\nu)_{ij} \bar{v}_{Li}^c v_{Lj} + \dots, \quad (2)$$

where the upper suffix c denotes the charge conjugation. The 4D Majorana neutrino masses $(m_\nu)_{ij}$ are given by

$$(m_\nu)_{ij} = \frac{v^2}{M_T} T(c_{li}) T(c_{lj}), \quad (3)$$

where $v \equiv e^{-\pi k R} \langle H \rangle \sim 100$ GeV is the VEV of the Higgs field, and $T(c)$ is

$$T(c) \sim \begin{cases} \sqrt{1/2 - c} & \text{for } c < 1/2, \\ \sqrt{c - 1/2} \times (M_T/M_P)^{c-1/2} & \text{for } c > 1/2 \end{cases} \quad (4)$$

(see Appendix A for details).

The 5D Yukawa interactions which generate 4D Dirac mass terms are

$$\int d^4x \int dy \sqrt{-g} \frac{(y_e)_{ij}}{M_P} H \bar{\Psi}_{li} \Psi_{ej} \delta(y - \pi R) \equiv \int d^4x (m_e)_{ij} \bar{e}_{Li} e_{Rj} + \dots, \quad (5)$$

where $(y_e)_{ij} \sim O(1)$ are dimensionless 5D Yukawa couplings. The 4D Dirac mass matrix $(m_e)_{ij}$ is given by

$$(m_e)_{ij} = v \times T(c_{li}) (y_e)_{ij} T(c_{ej}). \quad (6)$$

Using $m_\tau \sim (m_e)_{33} \sim 1$ GeV, we have

$$m_{\nu\tau} \sim (m_\nu)_{33} = \frac{v^2}{M_T} T(c_{l3}) T(c_{l3}) \sim \frac{v^2}{M_T} \left\{ \frac{m_\tau}{v(y_e)_{33} T(c_{e3})} \right\}^2 > 100 \text{ keV}, \quad (7)$$

where we have used $(y_e)_{33} \sim 1$ and $T(c) < 1$ for $c \sim 1$. This neutrino mass is well above the limit $\sum m_\nu < 0.68$ eV obtained from the WMAP observations [15] and the implications of atmospheric neutrino oscillation $m_{\nu 3}^2 \sim \Delta m_{\text{atm}}^2 \sim 10^{-3} \text{ eV}^2$ [16].

Dimension 6 operators

$$\hat{u}_{L1} \hat{d}_{L1} \hat{u}_{R1} \hat{e}_{R1}, \quad \hat{u}_{R1} \hat{d}_{R1} \hat{u}_{L1} \hat{e}_{L1}, \quad \dots, \quad (8)$$

lead to an unacceptably short proton lifetime, if they are not suppressed sufficiently. Here $\hat{u}_{L,R}, \hat{d}_{L,R}$ and $\hat{e}_{L,R}$ denote mass eigenstates of fermions. They are related to the electroweak eigenstates $u_{L,R}, d_{L,R}$ and $e_{L,R}$ via

$$\begin{aligned} u_{Li} &= \sum_j U_{ij}^L \hat{u}_{Lj}, & d_{Li} &= \sum_j D_{ij}^L \hat{d}_{Lj}, & e_{Li} &= \sum_j E_{ij}^L \hat{e}_{Lj}, \\ u_{Ri} &= \sum_j U_{ij}^R \hat{u}_{Rj}, & d_{Ri} &= \sum_j D_{ij}^R \hat{d}_{Rj}, & e_{Ri} &= \sum_j E_{ij}^R \hat{e}_{Rj}, \end{aligned} \quad (9)$$

where U^L , U^R , D^L , D^R , E^L and E^R are 3×3 unitary matrices. The eigenvalues of fermion mass matrices are

$$\begin{aligned}(m_u)_i &= v \times \sum_{j,k} (U^{R\dagger})_{ij} T(c_{uj})(y_u)_{jk} T(c_{qk}) U_{ki}^L, \\(m_d)_i &= v \times \sum_{j,k} (D^{R\dagger})_{ij} T(c_{dj})(y_d)_{jk} T(c_{qk}) D_{ki}^L, \\(m_e)_i &= v \times \sum_{j,k} (E^{R\dagger})_{ij} T(c_{ej})(y_e)_{jk} T(c_{lk}) E_{ki}^L,\end{aligned}\quad (10)$$

where dimensionless 5D Yukawa couplings y_u , y_d and y_e are $\sim O(1)$.

The operator $\hat{u}_{L1}\hat{d}_{L1}\hat{u}_{R1}\hat{e}_{R1}$ is a mixing of operators $u_{Lh}d_{Li}u_{Rj}e_{Rk}$. The effective suppression scales of the operators $u_{Lh}d_{Li}u_{Rj}e_{Rk}$ are

$$\frac{1}{M(u_{Lh}d_{Li}u_{Rj}e_{Rk})^2} = \frac{1}{M_T^2} T(c_{qh})T(c_{qi})T(c_{uj})T(c_{ek}) - \frac{1}{M_P^2} P(c_{qh})P(c_{qi})P(c_{uj})P(c_{ek}), \quad (11)$$

where

$$P(c) \sim \begin{cases} \sqrt{1/2 - c} \times (M_T/M_P)^{1/2-c} & \text{for } c < 1/2, \\ \sqrt{c - 1/2} & \text{for } c > 1/2 \end{cases} \quad (12)$$

(see [Appendix A](#) for details). Taking the mixing into account, the suppression scales of the operators $\hat{u}_{L1}\hat{d}_{L1}\hat{u}_{R1}\hat{e}_{R1}$ and $\hat{u}_{R1}\hat{d}_{R1}\hat{u}_{L1}\hat{e}_{L1}$ are

$$\begin{aligned}\frac{1}{M(\hat{u}_{L1}\hat{d}_{L1}\hat{u}_{R1}\hat{e}_{R1})^2} &= \sum_{h,i,j,k} \frac{U_{h1}^L D_{i1}^L U_{j1}^R E_{k1}^R}{M(u_{Lh}d_{Li}u_{Rj}e_{Rk})^2}, \\ \frac{1}{M(\hat{u}_{R1}\hat{d}_{R1}\hat{u}_{L1}\hat{e}_{L1})^2} &= \sum_{h,i,j,k} \frac{U_{h1}^R D_{i1}^R U_{j1}^L E_{k1}^L}{M(u_{Rh}d_{Ri}u_{Lj}e_{Lk})^2}.\end{aligned}\quad (13)$$

Using Eq. (11), we see that these summations are approximately given by³

$$\begin{aligned}\frac{1}{M(\hat{u}_{L1}\hat{d}_{L1}\hat{u}_{R1}\hat{e}_{R1})^2} &\simeq \sum_{h,i,j,k} \frac{1}{M_T^2} T(c_{qh})T(c_{qi})T(c_{uj})T(c_{ek}) U_{h1}^L D_{i1}^L U_{j1}^R E_{k1}^R, \\ \frac{1}{M(\hat{u}_{R1}\hat{d}_{R1}\hat{u}_{L1}\hat{e}_{L1})^2} &\simeq \sum_{h,i,j,k} \frac{1}{M_T^2} T(c_{uh})T(c_{di})T(c_{qj})T(c_{lk}) U_{h1}^R D_{i1}^R U_{j1}^L E_{k1}^L.\end{aligned}\quad (14)$$

Thus we have

$$\begin{aligned}\left[\frac{1}{M(\hat{u}_{L1}\hat{d}_{L1}\hat{u}_{R1}\hat{e}_{R1})^2} \right]^* \frac{1}{M(\hat{u}_{R1}\hat{d}_{R1}\hat{u}_{L1}\hat{e}_{L1})^2} &\simeq \frac{1}{M_T^2} \frac{1}{M_T^2} \sum_{h,h'} (U^{L\dagger})_{1h} T(c_{qh})T(c_{uh'}) U_{h'1}^R \sum_{i,i'} (D^{L\dagger})_{1i} T(c_{qi})T(c_{di'}) D_{i'1}^R \\ &\times \sum_{j,j'} (U^{R\dagger})_{1j} T(c_{uj})T(c_{qj'}) U_{j'1}^L \sum_{k,k'} (E^{R\dagger})_{1k} T(c_{ek})T(c_{lk'}) E_{k'1}^L.\end{aligned}\quad (15)$$

Since y_u , y_d and y_e are $\sim O(1)$ in Eq. (10), we have

$$\begin{aligned}\left[\frac{1}{M(\hat{u}_{L1}\hat{d}_{L1}\hat{u}_{R1}\hat{e}_{R1})^2} \right]^* \frac{1}{M(\hat{u}_{R1}\hat{d}_{R1}\hat{u}_{L1}\hat{e}_{L1})^2} &\sim \frac{1}{M_T^2} \frac{1}{M_T^2} \frac{(m_u)_1^*}{v} \frac{(m_d)_1^*}{v} \frac{(m_u)_1}{v} \frac{(m_e)_1}{v} \\ &\sim \frac{1}{(10^9 \text{ GeV})^4}.\end{aligned}\quad (16)$$

The decay rates of $p \rightarrow \pi^0 e^+$ induced by the dimension 6 operators $O_1 = \hat{u}_{R1}\hat{d}_{R1}\hat{u}_{L1}\hat{e}_{L1}$ and $O_2 = \hat{u}_{L1}\hat{d}_{L1}\hat{u}_{R1}\hat{e}_{R1}$ are

$$\Gamma(p \rightarrow \pi^0 e^+) = \sum_{i=1,2} \frac{1}{M(O_i)^4} \frac{1}{4\pi^2} \int \frac{d^3 q'}{2q_0} \int \frac{d^3 k'}{2k_0} |W(k - q')|^2 \frac{k'k}{2m_p} \delta^4(k - k' - q'), \quad (17)$$

³ There would be sets $\{i, j, k, l\}$ for which the second term in Eq. (11) dominates. However, contributions from these sets to the summation in Eq. (13) are negligible. If the second term in Eq. (11) dominates for all sets $\{i, j, k, l\}$, then the factors $T(c)$ are too small to explain the fermion masses.

where $W(k - q')$ is the form factor of the $p \rightarrow \pi$ matrix element

$$\langle \pi(q) | (\hat{d}_{R1} \hat{u}_{R1}) \hat{u}_{L1} | p(k) \rangle = W(q - k) p_L(k), \quad (18)$$

and k , k' and q are four momenta of proton, positron and pion, respectively. The momentum dependence of W is weak and $W \simeq -0.15 \text{ GeV}^2$ [17], so that

$$\tau(p \rightarrow \pi^0 e^+) \simeq 7.5 \times 10^{31} \times \left(\frac{\min[M(O_i)]}{10^{15} \text{ GeV}} \right)^4 \times \left(\frac{0.15 \text{ GeV}^2}{|W|} \right)^2 \text{ yr}. \quad (19)$$

Thus the suppression scales obtained in Eq. (16) are too small to explain the observed proton lifetime.

We have seen that operators concerning with the Majorana neutrino masses and the proton decay should be suppressed by small factors, or forbidden by some symmetries. We consider that the former solution contradicts with the philosophy of the RS model, that is, solving the hierarchy problem without fine tunings. In the next section, we look for the symmetries which may suppress these operators.

3. Discrete gauge symmetry

In this section, we look for the symmetries which may suppress the dangerous operators. We concentrate on gauge symmetries, since any global symmetries may be explicitly broken by the topological effects of gravity [14]. In addition, we consider discrete gauge symmetries, since there is no continuous anomaly-free symmetry except for $U(1)_{B-L}$, which cannot suppress the dangerous operators for the proton decay. We introduce only one discrete gauge symmetry: the gauge symmetry of the action is assumed to be $SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_N$, where $SU(3)_C \times SU(2)_L \times U(1)_Y$ is the gauge group of the Standard Model. Furthermore, we introduce 5D Dirac fields $\Psi_{\nu i}$ ($i = 1, 2, 3$) to obtain acceptable neutrino masses. The zero modes of $\Psi_{\nu i}$ are the right-handed neutrinos ν_{Ri} .

3.1. Z_N symmetry for the Dirac neutrino case

Here we discuss Z_N discrete gauge symmetries in the Dirac neutrino case. Those Z_N symmetries must respect the Yukawa terms. The Z_N charge of each field is constrained as Table 1. We can set the Z_N charge of the Higgs field to 0 without loss of generality by using a gauge rotation of $U(1)_Y$ in the Standard Model.

The anomaly cancellation conditions which include Z_N are

$$\begin{cases} 0 = \frac{1}{2} r_1 N, \\ \frac{9}{2} m + \frac{3}{2} p = \frac{1}{2} r_2 N, \\ 0 = \frac{\eta}{2} r_3 N + r_4 N, \end{cases} \quad (20)$$

where r_i are integers and $\eta = 1, 0$ for $N = \text{even, odd}$. The first equation comes from the cancellation of $\{Z_N\}\{SU(3)_C\}^2$ anomalies, the second from the cancellation of $\{Z_N\}\{SU(2)_L\}^2$ anomalies, and the last from the cancellation of Z_N -gravitational anomalies. Here we omit the cancellation of $\{Z_N\}^3$, $\{Z_N\}^2\{U(1)_Y\}$ and $\{Z_N\}\{U(1)_Y\}^2$ anomalies, because these constraints are always satisfied by adding heavy particles with appropriate charges [18,19]. There is an infinite number of $Z_N(p, m)$ discrete gauge symmetries which satisfy these constraints.

Now we examine which operators should be suppressed by symmetries. First, we consider operators with dimension $n \geq 12$. The proton lifetime derived from a dimension n operator is roughly estimated as

$$\tau_p \sim \frac{M_n^{2n-8}}{m_p^{2n-7}}, \quad (21)$$

where M_n is the suppression scale of the proton decay operator. For $n \geq 12$, τ_p is longer than the experimental bound $\tau_p > 1.9 \times 10^{29} \text{ yr}$ for $M_n = M_T = 10 \text{ TeV}$. The other B , L breaking operators with dimension $n \geq 12$ would also be sufficiently suppressed by M_T .⁴

Table 1
 Z_N charges consistent with the Yukawa interactions. Here we set the charge of the Higgs field to 0 by using a gauge rotation of $U(1)_Y$

	q_L	u_R	d_R	l_L	e_R	ν_R	H
Z_N	m	m	m	p	p	p	0

⁴ There may be operators with dimension $n \geq 12$ which require suppressions $M_n > 10 \text{ TeV}$. Even in that case, we can forbid them by choosing appropriate symmetries, or suppress them by tuning the fermion configurations. We can also suppress them by assuming that Z_N symmetries originate from $U(1)_X$ symmetries, as we will see in the next subsection.

Thus we concentrate on the B, L breaking operators with dimension $n < 12$, which include the dimension 6 proton decay operators and the dimension 5 neutrino mass operators discussed in Section 2. The Z_n charges of those operators are $2p, 4p, 6p, 3m \pm p, 3m \pm 3p$ and $6m$, and an infinite number of symmetries forbid these operators. For example, an infinite series of $Z_{9m+3}(1, m)$ ($m = 2, 3, \dots$) symmetries⁵ are appropriate. Thus, there is an infinite number of symmetries which forbid the dangerous operators in the Dirac neutrino case.

3.2. Z_N symmetry for the Majorana neutrino case

Let us consider the case where the seesaw mechanism [13] induces the light Majorana neutrino masses. Naively, this seems to be impossible, since the charges of $M\nu_R\nu_R$ are the same as those of HHl_Ll_L which induce too large neutrino masses. However, this is not the case, when the discrete gauge symmetries originate from $U(1)_X$ gauge symmetries.

Let us consider that the scalar field Φ which breaks the $U(1)_X$ symmetry lives on the Planck brane,⁶ and ν_R acquire the Majorana masses through the coupling with Φ :

$$\int d^4x \int dy \sqrt{-g} \frac{1}{M_P} \Phi \bar{\psi}_{\nu_i}^c \psi_{\nu_j} \delta(y) \equiv \int d^4x M_{ij} \bar{\nu}_{Ri}^c \nu_{Rj} + \dots, \quad (22)$$

where M_{ij} are given by

$$M_{ij} = \langle \Phi \rangle P(c_{\nu i}) P(c_{\nu j}). \quad (23)$$

Assuming $\langle \Phi \rangle \sim M_P$, M_{ij} take values between M_T and M_P for $0 < c_{\nu i}, c_{\nu j} < 1$.

The operators HHl_Ll_L also couple with Φ , since their $U(1)_X$ charges are the same as those of $\nu_R\nu_R$. Thus the dangerous Majorana neutrino mass terms appear only through combinations with Φ :

$$\int d^4x \int dy \sqrt{-g} \frac{1}{M_P^4} \Phi HH \bar{\psi}_{li}^c \psi_{lj} \delta(y) \delta(y - \pi R), \quad (24)$$

which should vanish, since H do not overlap with Φ .

Thus, the operators HHl_Ll_L induced directly from the Φ condensation are negligible. We now consider Yukawa interaction terms $H^\dagger \bar{l}_L \nu_R$ and the Majorana mass terms $M\nu_R\nu_R$ to estimate neutrino masses. A model with these mass terms was suggested in [21], and the effective light neutrino masses are approximately given by

$$(m_\nu)_{ij} = \frac{\{vT(c_{li})T(c_{\nu j})\}^2}{\langle \Phi \rangle P(c_{\nu j}) P(c_{\nu j})}, \quad (25)$$

which take values between ~ 1 GeV and $\sim 10^{-33}$ eV for $0 < c_{li}, c_{\nu j} < 1$. Thus observations concerning with neutrino masses are easily explained.

Let us count the Z_N discrete gauge symmetries which satisfy the anomaly cancellation conditions and respect the Majorana mass terms. The anomaly cancellation conditions are the same as those of the last subsection. Since the Majorana mass terms $M\nu_R\nu_R$ are induced from the interaction terms of Φ and ν_R , there arises another condition for p and N ,

$$2p = r_5 N, \quad (26)$$

where r_5 is an integer. Thus the discrete gauge symmetries are

$$\begin{aligned} Z_1(p, m) &= (0, 0): \text{ completely broken } U(1)_X, \\ Z_2(p, m) &= (1, 1), \\ Z_9(p, m) &= (0, 2), \\ Z_{18}(p, m) &= (9, 1). \end{aligned} \quad (27)$$

There are other discrete symmetries which satisfy these constraints. However, they are embedded in the above symmetries, or $U(1)_Y$ gauge equivalents of those [22].

All of these symmetries allow some of the dangerous operators, whose Z_N charges are $2p, 4p, 6p, 3m \pm p, 3m \pm 3p$ and $6m$. However, depending on the $U(1)_X$ charges of fermions, these operators can be suppressed. To see this, we analyze general properties of higher dimensional operators which include Φ .

⁵ The symmetries in this series are independent of each other for the following reason. Equivalent discrete gauge symmetries are related through the charge conjugation or $U(1)_Y$ gauge rotation. Under the convention of Table 1, any symmetry equivalent to $Z_N(p, m)$ takes the form $Z_{nN}(np, n(m + \frac{kN}{3}))$. Thus equivalent symmetries have a common value of N/p .

⁶ A model in which the lepton number symmetry is broken on the Planck brane for the Dirac neutrino case is discussed in [20].

Consider an operator which consists of $2n$ fermions and a scalar field Φ

$$\int d^4x \int dy \frac{1}{M_P^{4n-3}} \Phi \Psi_1 \Psi_2 \cdots \Psi_{2n} \delta(y) \equiv \int d^4x \frac{1}{M_{2n}^{3n-4}} \psi_1 \psi_2 \cdots \psi_{2n}, \quad (28)$$

where Ψ_i are 5D Dirac fields and ψ_i are their zero modes. Then the effective suppression scale M_{2n} is given by

$$\frac{1}{M_{2n}^{3n-4}} = \frac{1}{M_P^{3n-3}} \Phi P(c_1) P(c_2) \cdots P(c_{2n}) < \frac{1}{M_P^{3n-4}}, \quad (29)$$

where we used $\langle \Phi \rangle \sim M_P$ and Eq. (12). Thus any higher dimensional operator with nonzero $U(1)_X$ charge is Planck suppressed in 4D effective theory.

For all discrete symmetries in Eq. (27), we can set the $U(1)_X$ charges of fermions so that the dangerous operators have to couple with Φ . For example, consider the case where the $U(1)_X$ charges of Φ , q_L and l_L are 1, 3 and 2, respectively. The $U(1)_X$ symmetry is broken to $Z_1(0, 0)$ in low energy. In this case, all the dangerous operators have nonzero $U(1)_X$ charges, and become Planck suppressed operators.

We comment on $n - \bar{n}$ oscillation as a phenomenon which would distinguish between the above discrete gauge symmetries. The $n - \bar{n}$ oscillation is induced by the dimension 9 operator $(u_R d_R d_R)^2$. In the case of $Z_2(1, 1)$, $Z_9(0, 2)$ and $Z_{18}(9, 1)$ symmetries, this operator is always Planck suppressed, since u_R and d_R have nonzero $U(1)_X$ charges. In the case of completely broken $U(1)_X$, the $U(1)_X$ charge of $(u_R d_R d_R)^2$ can be set to zero. Then the suppression scale is determined by the configurations of quarks, and could be tuned to the current lower bound, which is evaluated to be 10^5 GeV [23]. Thus the $n - \bar{n}$ oscillation could be observed in future experiments,⁷ if the discrete gauge symmetry is $Z_1(0, 0)$.

4. Conclusion

It has been argued that short proton lifetime and too large neutrino masses are most likely induced in the RS model, if the hierarchical Yukawa matrices are explained only by the field configurations along the extra dimension. We have confirmed that the above unwanted phenomenon are inevitable, and hence searched for the discrete gauge symmetries which may forbid the dangerous operators in the Dirac neutrino case and the Majorana neutrino case. We have found that there is an infinite number of such symmetries for the Dirac neutrino case. For the Majorana neutrino case, the discrete gauge symmetries should originate from $U(1)_X$ gauge symmetries, and they should be broken on the Planck brane. Furthermore, if the $U(1)_X$ symmetry is completely broken, the $n - \bar{n}$ oscillation could be observed in future experiments.

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Appendix A. Profile of 5D fermion

Here we summarize the convention for fermions in the bulk [25]. The kinetic and mass terms of a 5D Dirac field Ψ are

$$\int d^4x \int dy \sqrt{-g} \{ \bar{\Psi} i \gamma^A e_A^M (\partial_M + \Omega_M) \Psi + m_D \bar{\Psi} \Psi \}, \quad (30)$$

where e_A^M is the vielbein, and Ω_M is the spin connection. The 5D Dirac mass m_D is odd, and parametrized as $m_D = c\sigma'$. The 5D Dirac field Ψ is decomposed to

$$\Psi(x, y) = \begin{bmatrix} \psi_L^{(n)}(x) f_L^{(n)}(y) \\ \psi_R^{(n)}(x) f_R^{(n)}(y) \end{bmatrix}, \quad (31)$$

where $\psi_L^{(n)}$ and $\psi_R^{(n)}$ are 4D Weyl spinors. A pair of Weyl spinors $(\psi_L^{(n)}, \psi_R^{(n)})$ forms a 4D Dirac spinor $\psi^{(n)}$ with mass $m_n \sim M_T$. The zero mode is chiral and does not have Dirac mass term, since the y direction is compactified on S^1/Z_2 . The chirality of zero mode depends on the Z_2 transformation property of Ψ . Here we take the zero mode to be left-handed. The wavefunction $f_L^{(0)}(y)$ for the chiral zero mode $\psi_L^{(0)}$ is

$$f_L^{(0)}(y) = N_0 e^{(2-c)k|y|}. \quad (32)$$

⁷ There are proposals to improve the precision constraints by at least two orders of magnitude [24].

The normalization condition is

$$\int dy e^{-3k|y|} f_L^{(0)}(y) f_L^{(0)}(y) = 1 \quad (33)$$

and we have

$$N_0 = \sqrt{\frac{(1-2c)k}{2\{e^{(1-2c)k\pi R} - 1\}}}. \quad (34)$$

$T(c)$ and $P(c)$ which we have used in this Letter are defined as

$$T(c) \equiv \frac{1}{\sqrt{k}} e^{-(3/2)k|y|} f_L^{(0)}(y) \Big|_{y=\pi R}, \quad (35)$$

$$P(c) \equiv \frac{1}{\sqrt{k}} e^{-(3/2)k|y|} f_L^{(0)}(y) \Big|_{y=0}. \quad (36)$$

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